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NON-LINEAR WAVE PROPAGATION

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13. ABSTRACT

This report is a summary of the work accomplished under THEMIS Project No. 65, "Non-Linear Wave Propagation." In addition it lists all technical reports, and the corresponding papers published in the course of the project.

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SUMMARY OF MAIN ACHIEVEMENTS

The deformation produced in non-linear layered media when subjected to dynamic loads has been analyzed. Specifically the concepts of non-linear impedance and reflection coefficients of an interface, separating different non-linear materials, has been used to obtain algorithms describing the decay of a pulse as it bounces back and forth in a slab of non-linear material contained between two other materials. Typically the deformation could be caused by the arrival of a shock of constant strength in one of the outer layers. It could also be caused when the slab, which is part of a composite material, impacts on some other non-linear elastic material. This study is possible because, for a wide range of model materials, whose constitutive equations approximate those of many real materials, the governing equations can be integrated.

A small amplitude finite rate theory governing resonant vibrations in crystals or in gas-filled tubes has been developed. In this theory the effect of energy radiated from the face of the crystal or from the end of the gas-filled tube has been taken into account. In the case of the gas-filled tube, good-agreement with experimental results has been obtained.

The mode of propagation of a long gravity wave of large amplitude as it moves into a region where the fluid is sheared and stratified in a vertical direction, has been analyzed in detail. Such waves occur in the atmosphere and the oceans. In the case when these waves occur in the atmosphere the predictions of the theory have been correlated with certain aspects of the actual behavior of severe storms. In the case when the waves occur in the ocean, the theory has been used to model the effects of strong currents on tsunamis.

The techniques used in classical geometrical acoustics have been extended to describe the propagation of large

amplitude waves through inhomogeneous media. This theory has been used to describe the motion of a plane acoustic pulse in the atmosphere, where the ambient stratification is due to the weight of the air. The theory has also been used to describe the propagation of a tsunami into shallow water.

The theory for the propagation of plane acoustic pulses has also been generalized to describe the propagation of non-planar waves. In particular, a non-linear ray theory for waves propagating in elastic materials has been constructed.

Theories describing the propagation of large-amplitude short pulses in viscoelastic materials have been developed. In particular, these provide techniques for the study of the propagation of weak shocks. Also, a theory has been developed which describes the deformation produced, in a non-linear viscoelastic material, by rapid cyclic loading at the boundary.

Work has been conducted on the existence of the de-excitation shocks (δ-shocks) in expanding non-equilibrium nozzle flows. Situations in which an adiabatic shock (α-shock) is embedded within the δ-shock were discussed. It was shown that the de-excitation process cannot lead to thermal choking. Numerical solutions of the full equations were also obtained. These solutions confirmed the existence of the δ-shocks. Possible asymptotic limiting states were deduced for a general class of rate equations.

An analysis has been carried out of the validity of the sudden-freeze approximation for supersonic nozzle flows of a vibrationally-relaxing gas, when the initial excitation level is finite. It was assumed, in agreement with experimental data, that the relaxation time changes exponentially with the translational temperature. It was found that, in this limit, the sudden-freeze method does provide a valid first approximation to the asymptotic frozen level. It was shown that the accuracy of the method can be improved by introducing a modified freezing point.

For supersonic nozzle flows of a condensible vapor, it is often observed that the non-equilibrium zone downstream of the condensation point is terminated by a sudden collapse of the supersaturated state. This fact has led to the concept of a condensation shock. Classical shock analyses do not predict either the shock position or the thermodynamic state immediately downstream of the shock, nor do they provide information on the conditions under which such shocks will occur. The work carried out in this area attempted to answer these questions for both heterogeneous and homogeneous nucleation. Conditions for the existence of such shocks were obtained in the high activation limit. It was found that the shock is preceded by a precursor zone whose structure is dominated by droplet production. A simple criterion for the shock position was derived. Further, it was shown that the asymptotic state downstream of the shock corresponds to a saturated state provided that the flow does not choke. Analysis of the full shock relations implied that thermal choking will not occur if the flow at the condensation is supersonic with respect to the frozen sound speed.

Work has also been carried out on the propagation of plane electromagnetic waves in a general isotropic, non-linear, non-dispersive medium. Formulae have been derived for the velocity of propagation of the shock in the presence of constant ambient electric and magnetic fields. It is found that the propagation velocities depend on the electric and magnetic fields immediately ahead of the shock and on their magnitudes immediately behind the shock.

Formulae have also been derived for the velocities of propagation of first-order electromagnetic discontinuities. It was found that there are, in general, two possible propagation velocities in a specified forward direction and correspondingly two possible polarization directions. Both the velocities and the directions of polarization depend on the magnetic induction field and electric displacement field immediately ahead of the

discontinuity. Expressions have also been obtained, in the case of plane discontinuities, for the manner in which the magnitude of the discontinuity changes as the surface of discontinuity propagates.

Simple wave theory has been applied to the determination of the manner in which harmonics of various orders grow as an initially sinusoidal wave propagates in a non-linear dielectric. This method is more powerful than those usually used, in that it enables us to calculate the amplitudes and phases of harmonics of all orders.

Our results agree well with those obtained by conventional methods for harmonics of low orders.

Work has been carried out on the propagation of electromagnetic waves in a non-linear isotropic material. It has been shown that small departures from the classical linear constitutive relations can have significant cumulative effects on the evolution of these waves. A non-linear far field theory has been developed. Propagation of weak electromagnetic shocks was also discussed.

Theories have been developed for photoelastic effects in dielectrics undergoing finite deformations in which the dielectric constant depends on the instantaneous values of the deformation gradients, or on their history. The latter theory, which may be expected to apply to viscoelastic materials, is developed both for solids and for fluids.

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